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Height-diameter relationships for Scots pine plantations in Bulgaria: optimal combination of model type and application

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Abstract. The height-diameter relationship is an important and extensively investigated forest model, but generalized and mixed-effects models of wider applicability are currently lacking in the forest modeling literature for Scots pine (*Pinus sylvestris* L.) plantations in Bulgaria. Considering the practical advantages of deterministic and mixed-effects models, the present study aims to derive a generalized deterministic height-diameter relationship and a simple mixed-effects model for plantation-grown Scots pine in Bulgaria. Ten generalized and six local models of adequate mathematical properties were selected and examined in several subsequent steps with a representative data set. A deterministic model was derived for tree height reconstruction from the individual tree diameters, stand dominant height and diameter, number of trees per hectare and stand age. Mixed-effects models were developed from the individual-tree and stand diameters and heights applicable to determine the height-diameter relationship in field surveys. Both types of models can be applied with confidence, according to their advantages and specifications, for estimating the height-diameter relationship of Scots pine plantations in Bulgaria, presenting a unique contribution for the particular species, study area and type of model. The choice of the tested models is relevant to the height-diameter relationship investigation of biologically related and geographically close species and types of stands and the study procedure allows repetition of the work to provide reliable solutions of the problem where information on such type of model is deficient or incomplete.
Keywords deterministic model, height-diameter relationship, mixed-effects model, model localization, practical model application, Scots pine plantations.

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Introduction

Height and diameter are among the most important tree characteristics and their relationship is not only used to characterize the vertical stand structure, but also is fundamental for elaborating and applying many growth and yield models. The height-diameter relationship has been the subject of numerous studies, resulting in the development of both local and generalized models (Temesgen & Gadow 2004, Lei et al. 2009), as well as purely deterministic (Schröder & Àlvarez González 2001, López Sánchez et al. 2003) and mixed-effects models (Calama & Montero 2004, Saunders & Wagner 2008, Crecente-Campo et al. 2010). Local height-diameter models adequately describe the relationship between both tree characteristics at stand level, if derived from a sufficiently representative sample of diameter-height measurements, and are often used in forest inventories. However, expansion of the predictions to a wider region would probably lead to biased predictions, as the relationship is highly dependent on the growth conditions and stand characteristics (López Sánchez et al. 2003). This has led to the elaboration of numerous generalized height-diameter models that include stand-level variables such as density, age, basal area, site index, and mean and dominant heights and diameters. As an alternative to the purely deterministic models, the mixed-effects models characterize the variability between different locations through the random components of the model parameters, which specific value for a given unit can be predicted if a supplementary sample of observations taken from that sampling unit is available (Castedo-Dorado et al. 2006). Thus, the generalized model form and the mixed-effects modeling approach can both be considered as different ways of localizing the height-diameter model to specific stands. Beyond the advantages and disadvantages of the different types of height-diameter models as regards their statistical properties and reliability, which are extensively discussed in the relevant literature (e.g., López Sánchez et al. 2003, Trincado et al. 2007), the applicability of the models to different practical situations must be evaluated. The generalized deterministic height-diameter model has the disadvantage of requiring information about stand level variables, which is not always available and the additional time and costs involved in obtaining such data may not always be justified. However, this type of model is the only option when the height distribution has to be reconstructed from stand-level variables, without any field survey to obtain supplementary height-diameter measurements. An accurate mixed-effects model based only on diameter measurements and subsample of heights will fit well to field conditions, but cannot be applied appropriately without a supplementary sample of observations for random-components prediction, unless such calibration is done through stand level variables rather than height measurements (which is not typical for mixed-effects models).

Scots pine (*Pinus sylvestris* L.) is a major tree species in the coniferous forests in Bulgaria, growing at altitudes between 800 and 2000 m above sea level and is widely distributed in the Bulgarian mountains, forming pure and mixed natural forests. Because of its relatively fast growth and good quality timber, as well as its ability to grow in harsh site conditions, it is one of the species most widely used for both erosion control afforestation and establishment of intensive plantations for timber production. The tree height-diameter relationship of the Scots pine plantations in Bulgaria has been explored through local models derived for scientific and inventory purposes on a small regional scale, and until now no generalized or mixed-effects models of wider applicability have been developed or proposed.

Considering the practical advantages of generalized deterministic and mixed-effects models, and the deficiency of unified height-diameter relationship for Scots pine plantations in

Bulgaria, the present study aims to fulfill two main objectives: (i) to derive a generalized deterministic height-diameter relationship that can be applied for tree height reconstruction from tree diameters and stand level variables; (ii) to propose a mixed-effects model based on stand and tree diameters and heights applicable for simple and reliable determination of the height-diameter relationship in field surveys.

Materials and methods

Data set

The data set used to fit the proposed height-diameter models was generated from both personally collected and published data records. The 266 variable-sized sample plots (Table 1) (of circular or rectangular form) from which the data were collected are situated in the mountains of Bulgaria, and cover the entire range of sites, densities and growth stages of the Scots pine plantations (Figure 1). One hundred and seven of these are temporary sample plots established and measured in 2002-2007. Data records from 33 permanent sample plots installed in 1-3 replications and measured 1-3 times were obtained from Forest Inventory Plans and other published data sources (Efremov 2006, Marinov 2008). In addition, 126 plot sets of height-diameter measurements used for developing growth and yield tables for Scots pine plantations in Bulgaria (Krastanov et al. 1980) were also included as part of the parameterization data set. Only plots that had not been thinned, or only thinned from below were included in the data set. As the sample plot designs differed, which was also reflected in the criteria used to select the tree subsample for height measurements, the total data set can be described as being composed of two principle subsets (Table 1). The subsample of measured tree heights in the first data subset was chosen to cover the entire range of tree diameters in the plot, while that of the second subset consisted of mean and dominant tree height measurements. In addition to the tree height-diameter data, the following stand level variables were also used in the analyses: stand density (ha^{-1}), dominant height (m), dominant diameter (cm), quadratic mean diameter (cm) and age (years) (Table 1).

Height-diameter models

In most local models (i.e. models that relate tree height to the breast-height diameter alone)

Table 1 Summary statistics for stand and tree variables, calculated from the subsets of data used to model the height-diameter relationship of Scots pine plantations in Bulgaria

Variable[a]		Subset 1 PM = 201, n = 2503				Subset 2 PM = 107, n = 1095			
		mean	standard deviation	Minimum	Maximum	mean	standard deviation	Minimum	Maximum
Stand level	D_d	22.6	6.0	12.5	34.5	22.6	8.0	6.7	43.1
	H_d	17.1	3.3	9.7	26.7	17.1	5.4	3.6	32.6
	D_m	17.9	4.7	8.2	27.7	16.8	6.8	2.5	35.3
	N	1983.0	1030.0	540.0	8050.0	2684.0	1682.0	613.0	8210.0
	A	37.0	9.0	18.0	70.0	37.0	14.0	10.0	78.0
	PS	1542.0	572.0	272.0	2000.0	280.0	186.0	85.0	700.0
Tree level	h	15.4	3.7	2.5	27.8	16.6	5.1	2.9	35.0
	d	17.8	6.6	2.0	41.2	20.0	7.9	4.0	47.0

Note. [a] Abbreviations used in the tables: D_d - dominant stand diameter (cm), H_d - dominant stand height (m), D_m - quadratic mean diameter (cm), N - stand density (ha^{-1}), A - stand age (years), h - tree height (m), d - diameter at breast height (cm), PM - total number of combinations of plot-measurement occasions, n - number of trees measured for heights, PS - plot size (m^2).

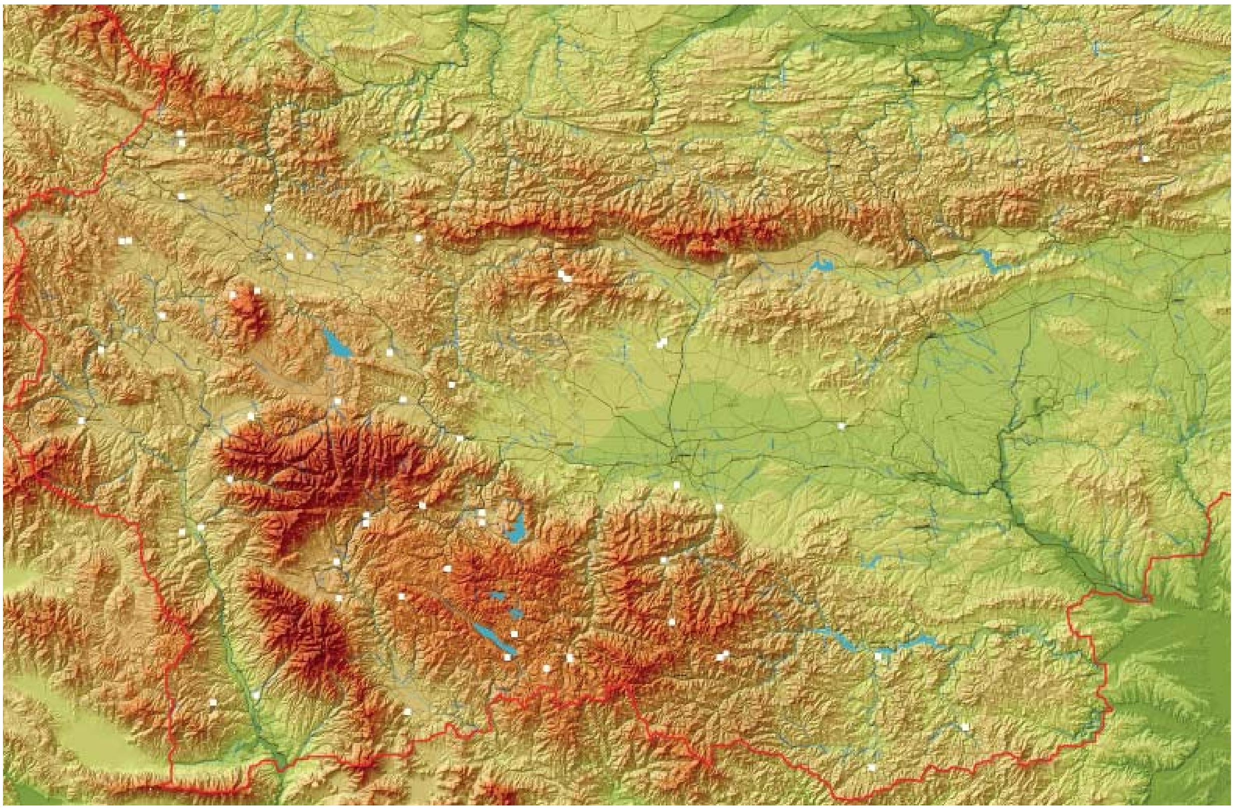

Figure 1 Indicative range of data collection in Scots pine plantations in Bulgaria. The central spots of the local Forest Ranges, on which territory the sample plots were established are marked by white squares

used to study the height-diameter relationships in Bulgarian forests, linear functions of the parameters have been tested (Dimitrov 2003). However, the non-linear equation form is generally preferred in this type of study worldwide (Calama & Montero 2004, Lei at al. 2009) and the scatter plot of the height-diameter data from the Scots pine plantations showed a typical sigmoid-curve pattern. Thus, six non-linear (L1–L6) functions of tree height on diameter were selected for evaluation in the present study (Table 2). Ten multiple regression equations of biologically consistent behavior and mathematical properties adequate to describe functionally the pattern revealed by the data were tested to describe tree height as a function of tree diameter and stand level variables (density, dominant height, dominant diameter, quadratic mean diameter and age) (Table 3). A relevant classification of the generalized types of height-diameter models, based on the degree of sampling effort (López Sánchez et al. 2003) was considered and height-diameter functions involving little sampling effort, i.e. measurement of all tree diameters and samples of tree heights, were preferred (G1–G7), although three models requiring data on stand age and density were also examined (G8–G10).

Model estimation, precision evaluation and comparison

Estimation, accuracy testing and comparison of the models were carried out according to the research objectives, in several subsequent steps. To fulfill the first aim of the study, the ten generalized models were fitted by the non-linear least squares method over the total data set, and the goodness-of-fit was assessed on the basis of the adjusted R^2 and Root Mean

Table 2 Local height-diameter models and goodness of fit statistics

Model[a]		Author	Mean bias (m)[b]	Aggregated MSE (m)	90[th] percentile of ARB_i
L1	$h = b_0 d^{b_1}$		-1.29×10^{-3}	203.91	8.45×10^{-3}
L2	$h = 1.3 + \exp(b_0 + b_1 / (d+1))$	Schreuder et al. (1979)	3.62×10^{-4}	195.84	7.62×10^{-3}
L3	$h = 1.3 + b_0 \exp(b_1 / d)$	Loetsch et al.(1973)	9.16×10^{-4}	196.67	7.56×10^{-3}
L4	$h = 1.3 + b_0 d^{b_1}$	Stage (1975)	-1.80×10^{-3}	205.18	8.59×10^{-3}
L5	$h = 1.3 + b_0 d / (b_1 + d)$	Wykoff et al. (1982)	-2.10×10^{-3}	199.20	8.50×10^{-3}
L6	$h = 1.3 + b_0 \left[1 - \exp\left(-b_1 d\right)\right]^{b_2}$	Peng et al. (2001)	3.03×10^{-4}	229.00	8.85×10^{-3}

Note: [a] Abbreviations: h – tree height (m), d – diameter at breast height (cm); [b] Formulae: Mean bias – $\frac{\sum_{i=1}^{PM} bias_i}{PM}$; Aggregated MSE – mean square errors aggregated from all subjects – $\sum_{i=1}^{PM}(bias_i^2 + \mathrm{var}_i)$; 90th percentile of ARB_i (absolute relative biases), where $bias_i$ is the mean error per subject $bias_i = \sum_{j=1}^{N}\frac{(\hat{h}_j - h_j)}{N}$, var_i is the error variance per subject, $\mathrm{var}_i = \sum_{j=1}^{N}\frac{(\hat{h}_j - h_j)^2}{N}$, ARB_i is the absolute relative bias per subject $ARB_i = \left|\frac{\sum_{j=1}^{N}\frac{(\hat{h}_j - h_j)}{h_j}}{N}\right|$, PM – total number of subjects, N – number of the measured tree heights per subject, h_j – observed tree height, $\hat{h}_j$ – predicted tree height.

Square Error ($RMSE$) of regressions, the model bias, the actual vs. predicted values analysis (Table 3). Visual examination of the residual plots (independent variables vs. residuals, predicted height values vs. residuals) and modified Breusch-Pagan test were applied to check for heteroscedasticity of the errors. The best generalized deterministic model was chosen for tree height distribution reconstruction from tree diameters and stand level variables. The best generalized model including only height and diameter predictor variables (i.e. tree diameter, dominant height, stand quadratic mean or dominant diameters) was selected for further comparison for the second research objective.

To fulfill this objective, the six local models were parameterized by the subjects (i.e. combination of plot and measurement occasion) included in the total data set (i.e. 308 subjects). Several test statistics were used to evaluate and compare the model performance: the mean bias estimated as an average of the different subject biases (Calama & Montero 2004), mean square error (MSE) calculated per subject and aggregated to a single value for each model, and the value of the 90th percentile of the absolute relative biases evaluated for the subjects (Table 2). These three characteristics were chosen for assessment of both the bias and precision of the local models, as tested for a broad range of growth performances, taking into account the average (mean bias, aggregated MSE), as well as the range (aggregated MSE, 90th percentile of the relative bias) of model errors. The local models were also screened for their general applicability to the data by fitting the models over the entire data set and examining the principal test statistics of the regression analysis as well as by exploring the residual plots and the plot of actual vs. predicted values for the regression models fitted by subjects.

The best performing local model and the

Table 3 Generalized height-diameter models and goodness of fit statistics

Model [a]	Author	[b] *Adj.* R^2	*RMSE* (m)	Bias (m)	$\hat{h} = ah$ [c]	Parameter estimate: Parameter		Standard error: OLS covariance matrix	Standard error: HCCME
G1 $h=\left[1.3^{b_0}+(H_d{}^{b_0}-1.3^{b_0})\dfrac{1-\exp(-b_1 d)}{1-\exp(-b_1 D_d}\right]^{(1/b_0)}$	Shnute (1981)	0.902	1.399	-1.4×10^{-2}	0.993	b_0	0.09062	0.00186	0.00209
						b_1	1.00430	0.00080	0.00080
G2 $h=1.3+\left[b_0\left(\dfrac{1}{d}-\dfrac{1}{D_d}\right)+\left(\dfrac{1}{H_d-1.3}\right)^{\frac{1}{3}}\right]^{-3}$	Mønness (1982)	0.904	1.386	-5.6×10^{-2}	0.993	b_0	0.95233	0.01390	0.01630
G3 $h=1.3+\left[b_0\left(\dfrac{1}{d}-\dfrac{1}{D_d}\right)+\left(\dfrac{1}{H_d-1.3}\right)^{\frac{1}{2}}\right]^{-2}$	Cañadas et al. (1999) I	0.906	1.372	-5.6×10^{-2}	0.993	b_0	0.91297	0.01340	0.01560
G4 $h=1.3+(H_d-1.3)\left(\dfrac{1-\exp(b_0 d)}{1-\exp(b_0 D_d)}\right)$	Cañadas et al. (1999) III	0.901	1.407	-4.9×10^{-3}	0.993	b_0	-0.08392	0.00134	0.00155
G5 $h=1.3+(H_d-1.3)\exp\left(b_0\left(\dfrac{1}{D_d}-\dfrac{1}{d}\right)\right)$	Gaffrey (1988) modified by Diéguez Aranda et al. (2006)	0.898	1.429	-6.6×10^{-2}	0.993	b_0	6.90950	0.10090	0.12130
G6 $h=b_0 H_d\left(1-\exp\left(\dfrac{-b_1 d}{D_m}\right)\right)^{b_2}$	Pienaar et al. (1991)	0.879	1.468	-3.1×10^{-3}	0.992	b_0	1.09610	0.01130	0.01490
						b_1	1.79960	0.11570	0.15260
						b_2	1.06700	0.07630	0.09990
G7 $h=1.3+\exp\left(b_0+b_1 H_d+\dfrac{b_2 D_d}{d+1}\right)$	Tomé (1989) Mod.	0.867	1.628	3.9×10^{-2}	0.990	b_0	2.08100	0.01340	0.01660
						b_1	0.05606	0.00047	0.00075
						b_2	-0.33003	0.00763	0.00805
G8 $h=H_d\exp\left(\left(b_0+b_1 H_d+b_2\dfrac{N}{1000}+Ab_3\right)\left(\dfrac{1}{d}-\dfrac{1}{D_d}\right)\right)$	Tomé (1989)	0.906	1.333	-1.0×10^{-1}	0.994	b_0	-1.13530	0.70260	0.67470
						b_1	-0.41555	0.04890	0.05960
						b_2	0.31985	0.08990	0.08080
						b_3	0.04085	0.01700	0.02010

Table 3 (continuation)

1		2	3	4	5	6	7		8	9
G9	$h=\dfrac{H_d}{\exp\left(b_0+\left(\dfrac{1}{d}-\dfrac{1}{D_d}\right)\left(b_1+b_2\ln N+b_3\dfrac{1}{A}+b_4\ln H_d\right)\right)}$	Lenhart (1968) modified	0.907	1.329	9.7 × 10^{-3}	0.994	b_0	0.0116	0.0021	0.0023
							b_1	-10.196	3.876	4.136
							b_2	-0.9073	0.2133	0.2010
							b_3	82.742	18.979	19.915
							b_4	7.2384	0.8734	0.9874
G10	$h=b_0H_d^{b_1}10^{\left(\frac{b_2}{A}+\left(\frac{1}{d}-\frac{1}{D_d}\right)\left(b_3+\frac{b_4\log N}{A}\right)\right)}$	Amateis et al. (1995) modified	0.904	1.349	-4.8 × 10^{-5}	0.994	b_1	1.3351	0.0639	0.0683
							b_0	0.9087	0.0134	0.0147
							b_2	-0.5560	0.1759	0.1745
							b_3	-3.4327	0.1034	0.1073
							b_4	9.2587	1.0276	0.9113

Note. [a]Abbreviations: h – tree height (m), d – diameter at breast height (cm), D_d – dominant stand diameter (cm), H_d – dominant stand height (m), D_m – quadratic mean diameter (cm), N – stand density (ha^{-1}), A – stand age (years), OLS – Ordinary Least Squares, HCCME – Heteroscedasticity Consistent Covariance Matrix Estimate.

[b]Formulae: Bias (m) = $\dfrac{\sum_{i=1}^{n}(\hat{h}_i-h_i)}{n}$; Root mean square error (m) – RMSE = $\sqrt{\sum_{i=1}^{n}\dfrac{(\hat{h}_i-h_i)^2}{n-p}}$; Adjusted $R^2 = 1-\dfrac{(n-1)\sum_{i=1}^{n}(\hat{h}_i-h_i)^2}{(n-p)\sum_{i=1}^{n}(h_i-\bar{h})^2}$, where h_i is observed, $\hat{h}_i$ - predicted and $\bar{h}$ - mean observed height; n is number of height measurements, p is number of parameters. [c] Adj. R^2 of linear regression through the origin for the predicted on the actual height values

best generalized model including only height and diameter predictor variables were fitted as mixed models, by using the maximum likelihood method in the NLMIXED procedure of SAS/STAT (2002). No clear pattern of temporal or spatial correlation between the different measurements could be expected, because a substantial part of the data was cross-sectional, with most of the plots situated at long distances from each other. In addition, few-measurements (1-3) of the permanent sample plots were made, and the heights of different sub-samples of trees were usually measured each time. Thus, the inclusion of random parameters for the tree and temporal (measurement-occasion) level was not considered and the mixed-effects model structure was implicitly derived following the procedure described by Castedo-Dorado et al. (2006) and Trincado et al. (2007), and summarized in Table 4. The mixed-effects models performance was assessed and compared using the estimates of

Table 4 Parameter estimates and goodness of fit statistics for the local (L2) and the generalized (G3) mixed-effects models

Model abbreviation[a]		L2		G3	
Mixed-effects model formulation $h_i = f\ (x_i, \Phi_i)\ +\ \varepsilon_i$ $\Phi_i\ =\ F_i\beta\ +\ R_i s_i$		$h = 1.3 + \exp\left(b_0 + u + \frac{b_1 + v}{d+1}\right)$	$x_i = \left(d_{ij}\right)$	$h = 1.3 + \left[(b_0 + u)\left(\frac{1}{d} - \frac{1}{Dd}\right) + \sqrt{\frac{1}{Hd - 1.3}}\right]^{-2}$	$x_i = \begin{pmatrix} d_{ij} \\ Dd_i \\ Hd_i \end{pmatrix}$
Fixed parameters[b] β $(p \times 1)$	b_0	3.057(0.016)	$\beta = \begin{pmatrix} b_0 \\ b_1 \end{pmatrix}$	0.876 (0.033)	$\beta = (b_0)$
	b_1	-7.795 (0.220)			
Variance components[b]	σ_u^2	0.063 (0.006)		0.112 (0.020)	
	σ_v^2	9.311 (1.202)			
$\begin{pmatrix} \mathbf{s_i} \\ \boldsymbol{\varepsilon_i} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbf{D} & 0 \\ 0 & \mathbf{R}_i \end{pmatrix}\right)$	σ_{uv}	-0.469 (0.075)	$D = \begin{bmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{bmatrix}$		$D = \left[\sigma_u^2\right]$
D=Var[s_i], R_i=Var[ε_i]. $R_i = I_i\sigma^2$	σ^2	1.058 (0.027)		1.576 (0.046)	

[c] Model performance	L2	G3
RMSE	0.957	1.229
Bias	-0.028	-0.098
-2LL[d]	12116.83 (5457.59, $p < 0.001$)	8324.91 (241.15, $p < 0.001$)
AIC	12116.83	8330.91

Note: [a] Abbreviations: *Dd* – dominant stand diameter (cm), *Hd* – dominant stand height (m), *d* - diameter at breast height (cm); RMSE – root mean square error, -2LL = -2 × logarithm of likelihood function, AIC – Akaike's Information Criterion= -2LL + 2*r*, *r* – number of parameters; $\mathbf{x_i}$ – vector of independent (tree and stand dimension) variables; Φ_i – parameter vector ($r \times 1$); F_i, R_i – design matrices of size $r \times p$ and $q \times r$, for fixed and random effects specific for each subject, respectively; *i* - *i*-th subject, *i* (1 ÷ PM); *j* – *j*-th tree within *i*-th subject; β - ($p \times 1$) vector of fixed effects and b_0, b_1 – fixed parameter components, s_i = (q×1) vector of random effects and *u, v* – random parameter components, σ_u^2, σ_v^2, σ_{uv} – variances and the covariance of the random components; D=Var[s_i] – variance-covariance matrix of the random effects, R_i = Var[ε_i] - within-subject variance – covariance structure; σ^2 – residual variance of the model; I_i – identity matrix; ε_i = random error.
[b] Standard errors are shown in brackets.
[c] The formulae for Bias, RMSE are as in Table 3.
[d] The Likelihood Ratio Test Statistics and their significance are shown in brackets.

Bias, *RMSE*, *-2LL* (-2×logarithm of likelihood function) test statistics and Akaike's Information Criterion (Table 4). The Likelihood Ratio Test (Table 4) was performed to provide information about the degree of improvement of the model fit by inclusion of random parameters relative to the models with the fixed effects alone (Fang & Bailey 2001).

All formulae applied in the models estimation, precision evaluation and comparison are displayed below Tables 2, 3 and 4.

Results

Estimations of the local and generalized regression models are shown in Tables 2 and 3. Equation G7 produced residual and actual vs. predicted values plots indicating model inadequacy and was excluded from further comparisons on the basis of analytical test statistics. Coefficient b_0 in Model G8 was not statistically significant (Table 3) and was subsequently omitted from the regression, but this did not substantially improve the other goodness of fit statistics of the regression. The graphical and analytical tests indicated presence of heteroscedasticity of errors (Figure 2) for all tested models, which imposed additional application of Heteroscedasticity-Consistent Covariance Matrix Estimation (HCCME) (Long and Ervin, 2000) to assure the efficiency of the regression estimates (Table 3). Model G9, proposed by Lenhart (1968) and applied in a modified form in the present study, was the most adequate generalized height-diameter model of the 10 regressions tested (Table 3). It was chosen for reconstruction of the tree heights from the tree diameter and stand-level variables dominant height and dominant diameter, stand age and density in fulfillment of the first study objective. Model G3 (Cañadas et al. 1999) displayed the best regression properties among the generalized models including only height and diameter predictor variables, as indicated by the test statistics (Table 3), and was selected for further analysis.

The local models fitted successfully the height-diameter relationship in 288 (197 from subset 1, and 91 from subset 2) out of the 308 plot-measurement occasion combinations and Figure 3 illustrates the fitness of the tested equations to model the trend of the height - diameter data. When fitted to all data, two of the three model parameters of L6 were not significant. Of the other five local models, model L2 developed by Schreuder et al. (1979) was the most suitable for describing the height-diameter relationship of the Scots pine plantations studied, as indicated by the low bias and the relatively narrow range of errors (Table 2).

All parameters in models L2 and G3 were assumed to be composed of both fixed and random parts, and when fitted as mixed models, resulted in low error estimates and statistically significant fixed coefficients and variance components (Table 4). The Likelihood Ratio Test revealed a substantial increase in the goodness of fit by expansion of the parameters with random parts, which, together with the significance of all variance components of both models, suggested that the full mixed-effects models should be considered in a subsequent calibration of the random parts. The lower values of the -2LL test statistics and the Akaike's information criterion, usually used to compare models with alternative sets of fixed-effects and covariance parameters, inferred the superiority of the generalized mixed-effects model (G3) as compared to the local-one (L2). The values of Bias and RMSE, closer to the optimal values of 0 for the local model (Table 4), on the other hand, suggested that the difference in the goodness of fit could still be obscured when the random components are predicted by subsample of additional measurements (Figure 4). Thus, examination of different calibration approaches with supplementary observations, when appropriate data are available, would be relevant for final selection of mixed-effects model, together with optimal calibration of the random components through additional height measurements.

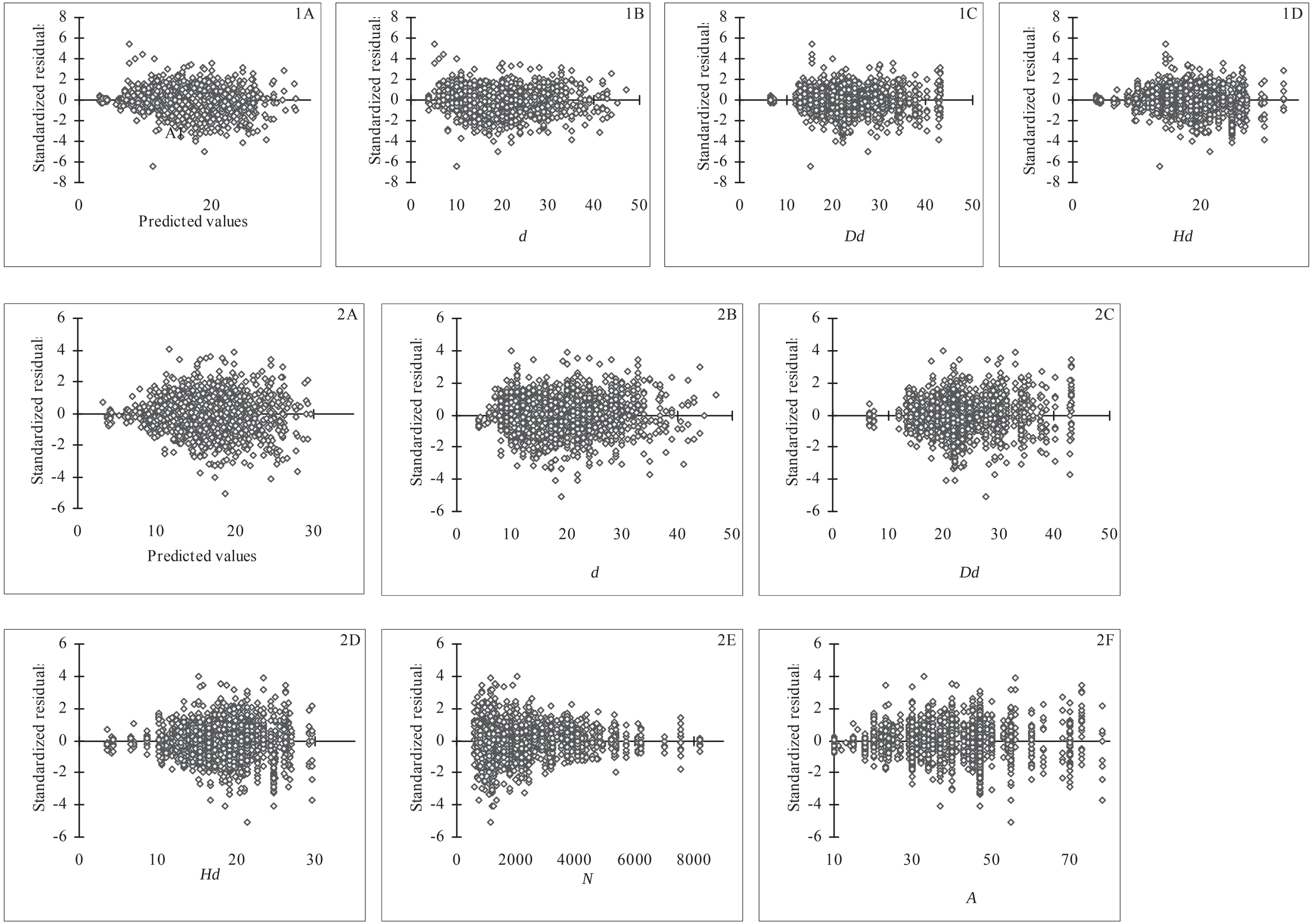

Figure 2 Plots of errors (predicted values and independent variables vs. residuals) for the two selected generalized height-diameter relationships. G3: 1A - 1D; G9: 2A - 2F. Abbreviations: d – diameter at breast height (cm), D_d – dominant stand diameter (cm), H_d – dominant stand height (m), N – stand density (ha^{-1}), A – stand age (years)

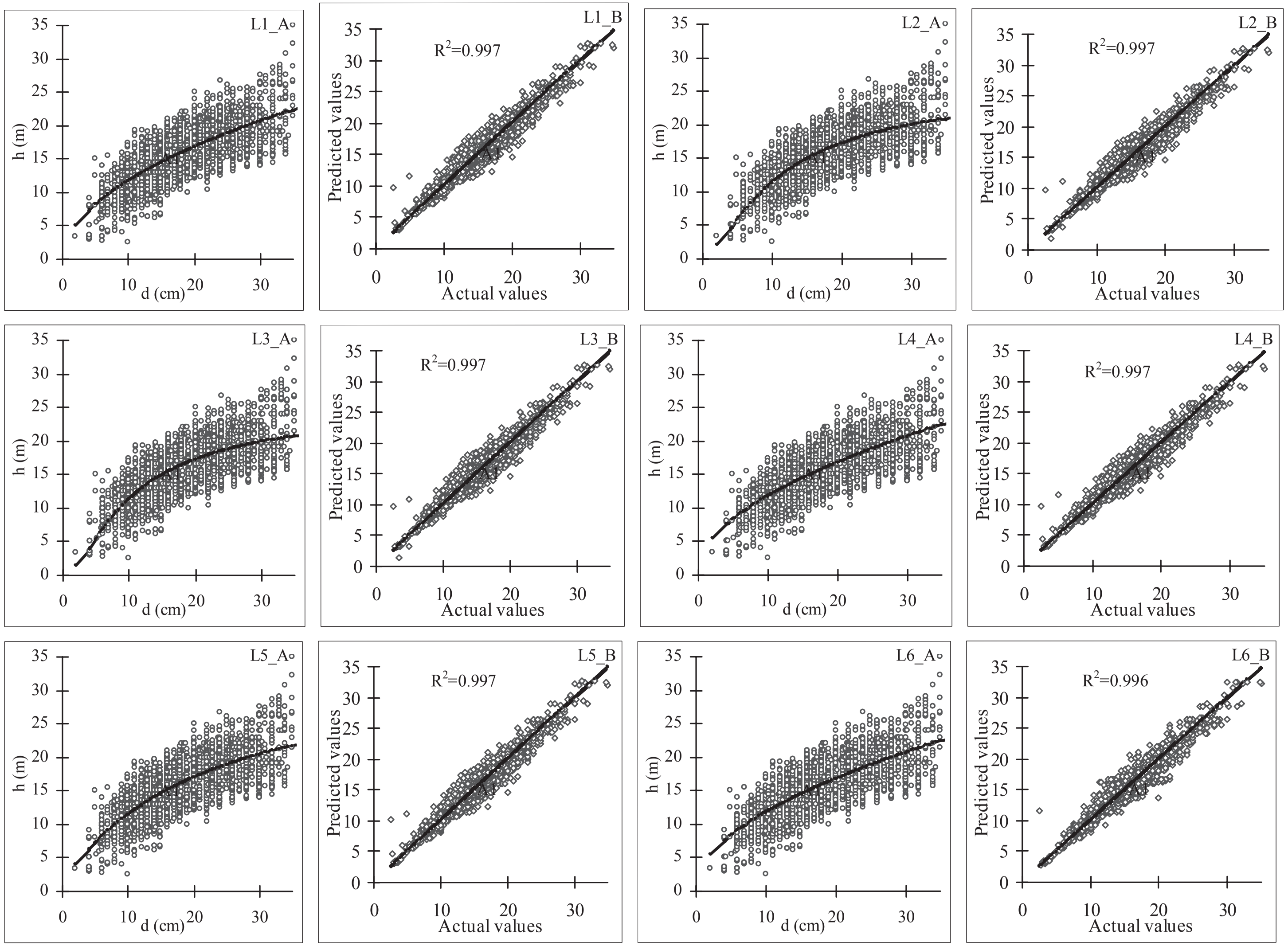

Figure 3 Graphical screening of the local models for their general applicability to the data. A .General trend of the local model L(1 - 6) compared to the experimental data; B . Plots of actual vs. predicted height values, fitted by subjects. Abbreviations: h - tree height (m), d - diameter at breast height (cm)

Discussion

The present study aimed to develop two alternatives to the local height-diameter models derived for individual stands, which combine precision and simplicity, but are applicable in different situations. The first alternative derived is a deterministic generalized model for Scots pine plantations in Bulgaria, which can be applied for reconstruction of tree height distribution from tree diameters and stand level variables. The second option developed and proposed in the current study is a mixed-effects model based only on height and diameter predictor variables for simple and reliable determination of the height-diameter relationship in field surveys. A representative data set composed of temporary and permanent sample plots and covering the entire range of distribution and variety of plantations was used for parameterization and comparison of the candidate models.

The model derived in response to the first research objective was adequate, as revealed by the goodness of fit statistics and trends in the predictions, and its biologically sound formulation. The predictions of the deterministic generalized height-diameter model G9 are based on the stand level variables dominant height, dominant diameter, stand density and age, all of which have a notable influence on the studied relationship and its dynamics. The maximum diameter in the original model form (Lenhart 1968) was replaced by the dominant diameter, to avoid the susceptibility of the plot values of the maximum diameter to outlying observations. Together with the dominant height, which is closely related to site quality, the dominant diameter enables adjustment of the estimated relationship to the stand growth rate and yield potential. Stand age and density are also assumed to be predictors of the height-diameter relationship, because the larger number of trees per unit area presupposes smaller diameters of stands of the same growth stage; stand age, on the other hand, reflects the differing temporal changes in the growth of the two variables. The present findings are consistent with those of previous studies, i.e. that inclusion of stand characteristics improves model accuracy regarding tree height predictions (López Sánchez et al. 2003, Lei et al. 2009), which was also observed from comparison with the goodness of fit of the simplified generalized models (Table 3, Figure 4).

The Scots pine is a mountainous tree species at southern latitudes, and in Bulgaria plantations are usually established on slopes of different inclination, on poor sites and very often planted at high initial densities, for the purpose of erosion control. As a result of the difficult sampling conditions, height measurements are limited to a relatively small subsample of trees, and the derived local model is usually applied to determine the mean stand height but is rather unreliable for reconstruction of the stand height distribution. The present study developed mixed-effects models, appropriate for localization of height-diameter relationships in field surveys, since it involves measurement of tree diameters and a small subsample of heights, while also providing reliable prediction of the tree height structure at stand level. The models tested for development of the mixed-effects approach only included diameter and height variables, but the number of trees per hectare, easily determined in field surveys, can also be considered as a covariate in future investigations, as suggested in a study by Calama & Montero (2004). The application of either the local (L2) or the generalized (G3) mixed-effects model selected in the present study, would bring a significant improvement in the predictability of the relationship as compared to the same models fitted as purely deterministic (Figure 4), and consequently will produce much better goodness of fit as compared to any of the local models. Calibration of the random parts of the models could be experimented with samples of at least 1 additional tree height, as suggested by Trincado et al. (2007), while having in mind the general

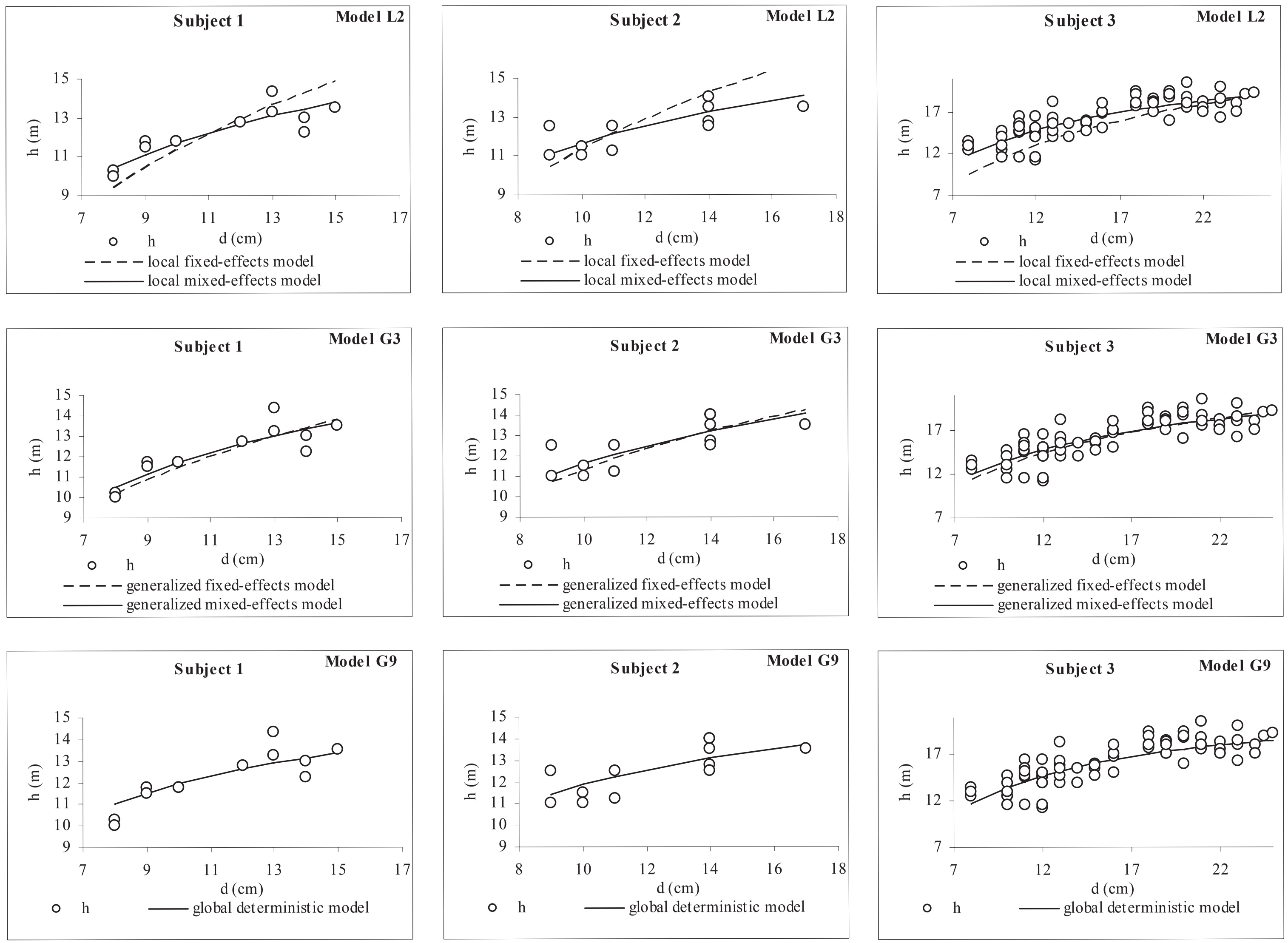

Figure 4 Goodness-of-fit of the selected height-diameter models illustrated with data for 3 subjects (plot-measurement occasion combinations). Abbreviations: *h* - tree height (m), *d* - diameter at breast height (cm)

tendency of improved precision with increased number of trees for calibration (Castedo-Dorado et al. 2006) and in consideration of the finding by Subedi & Sharma (2011), according to which the calibration with random and average-sized trees provide smaller biases.

Conclusions

Both types of models derived here can be applied with confidence, according to their advantages and specifications, for estimating the height-diameter relationship of Scots pine plantations in Bulgaria, presenting a unique contribution for the particular species, study area and type of model, currently lacking in the forest modeling literature. The choice of the tested models is relevant to the height-diameter relationship investigation of biologically related and geographically close species and types of stands and the study procedure allows repetition of the work to provide reliable solutions of the problem where information on such type of model is deficient or incomplete.

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